

Vector Field Based Approach to Reconstruct and Represent Functional Spaces with Areal Cartogram

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Summary

This study adopts the vector field approach to tackle the problem posed by asymmetric distances during the reconstruction of a functional space from empirical spatial interaction data. An interaction field can be estimated by linking the vector concept with inverted doubly constrained spatial interaction models. It enables to infer a potential field, as a forcing function giving rise to the asymmetry of spatial interaction. With the potential field, a density-equalizing areal cartogram is applied to map a global representation of the reconstructed functional space. Observed migration flows between U.S. states are used to demonstrate the validity of our approach.

KEYWORDS: functional space, spatial interaction, vector field, areal cartogram, migration

1. Introduction

The conceptualization of space is of great importance to the understanding of the spatial interaction processes that shape geographic phenomena. Functional spaces are tightly connected to human perception and utilization of the physical space or to the modalities of functional relationships between places embedded in the physical space. In his 1984 study of human migrations in the U.S., Plane (1984) set to elicit the space defined by functional relationships, after he noted that absolute physical distances usually provide an inferior measure of the relative functional distances that affect destination choices of US interstate migrants. He proposed to use inverted doubly constrained models with observed interstate migration flows as input to calibrate and estimate the cognized distances between states. As a result, a local view of the functional ‘migration space’ surrounding each origin and destination can be reconstructed and visualized as a distorted map through linear cartogram. However, this approach may end up with too many local views due to a large number of origins/destinations. Therefore, how to synthesize all these local views to form a global view of the functional space becomes an issue.

Spatial interactions between geographical areas are often and conveniently measured by “origin-destination” matrices, usually with asymmetric entries. This would result in asymmetric functional or cognitive distances that are inferred from empirical migration flows in Plane (1984)’s approach. Time distance is another example of asymmetric functional distances (Ahmed and Miller, 2007), where travel times, as the distance metric between two locations, are different according to direction. A major approach in the literature to model and visualize functional distances is called multidimensional scaling (MDS), which transforms relative distances between locations into an absolute space depicting these locations with coordinates (Chino, 2011). With the capacity to account for asymmetric

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distances, the visual representation of MDS techniques is in essence in line with distance-based linear cartograms. Another approach is, as proposed by Tobler (1967), to apply vector field to elicit and depict the implicit forces in interactions. Although it does not transform relative distances directly into a functional space, the vector field approach provides an alternative perspective to synthesize spatial interaction patterns, which can facilitate the creation of a global view of functional space.

This study proposes to employ the vector field approach to tackle the problem posed by asymmetric distances in the reconstruction of the functional space as a synthesis of the flow data. A vector-based interaction field can be interpreted as facilitating interaction flows in particular directions. The components of the vector field can be estimated within the framework of reversed doubly constrained spatial interaction models. Based on the interaction field, it is possible to infer a scalar potential as a forcing function that explains the observed interaction pattern. This derived forcing function presents a new possibility to visualize the reconstructed functional space as a continuous areal cartogram. As a result, a global view of the functional space can be created with a density-equalizing cartogram approach. Observed U.S. interstate migration flows will be used here to demonstrate the validity of our methodology.

2. Methodology

2.1. Theoretical models

To apply the vector field approach, we first make the theoretical connection between the model in Plane (1984) and that in Tobler (1967). In Plane (1984), a doubly constrained migration model is written as:

$$m_{ij} = A_i O_i B_j I_j d_{ij}^{-\gamma} \epsilon_{ij} \quad (i, j = 1, \dots, n; i \neq j) \quad (1)$$

where m_{ij} is the observed the migration flows from region i to region j , O_i is the out-migrants from region i , I_j is the in-migrants to region j , d_{ij} is the physical distance between regions i and j , γ is the distance-decay parameter, A_i and B_j are balancing factors, and ϵ_{ij} is the multiplicative stochastic error term. To fit a linear regression model, solving $\ln d_{ij}$ gives:

$$\ln d_{ij} = \frac{1}{\gamma} [\ln A_i + \ln B_j + \ln(O_i I_j / m_{ij})] + \frac{1}{\gamma} \ln \epsilon_{ij} \quad (2)$$

where the new error term $\frac{1}{\gamma} \ln \epsilon_{ij}$ encompass any specification error and any measurement errors associated with the observed interregional flows, as well as pure random effects. Therefore, the inferred distance estimates will be:

$$\ln \hat{d}_{ij} = \frac{1}{\gamma} [\ln A_i + \ln B_j + \ln(O_i I_j / m_{ij})] \quad (3)$$

Insert Equation 3 into Equation 2, the ratio of physical to inferred distances can be obtained as the transformed error term:

$$\frac{d_{ij}}{\hat{d}_{ij}} = \epsilon_{ij}^{\frac{1}{\gamma}} \quad (4)$$

which is consistent with Tobler (1967)'s basic assumption (Equation 5) about a flow vector \vec{c}_{ij} facilitating the travel efforts (interpreted as perceived or inferred distances) from region i to j regarding the functional form.

$$\frac{d_{ij}}{\hat{d}_{ij}} = r + \vec{c}_{ij} \quad (5)$$

where r is a constant rate of travel. Interpretations of \vec{c}_{ij} might be transportation facilities or social connections. Along this line, modelling \vec{c}_{ij} is equivalent to modelling $\epsilon_{ij}^{\frac{1}{r}}$, since their relationship can be developed as:

$$\vec{c}_{ij} = \frac{\epsilon_{ij}^{\frac{1}{r}} - \epsilon_{ji}^{\frac{1}{r}}}{2} \quad (6)$$

Therefore, it is appropriate to apply the vector field approach to the transformed error terms $\epsilon_{ij}^{\frac{1}{r}}$ for the reconstruction of the functional space as a synthesis of the flow data. Following Tobler (1967), a vector field can be obtained by summing up flow vectors for each region:

$$\vec{c}_i = \frac{1}{n-1} \sum_{j=1, j \neq i}^n \vec{c}_{ij} \quad (7)$$

2.2. Procedural methods

The proposed methodology to reconstruct functional space through density-equalizing areal cartogram can be described as a procedural approach, which is illustrated in Figure 1. Spatial interaction measured by observed migration flows is first represented by interaction vectors based on the proposed theoretical model. An interaction vector field can be further developed by spatial interpolation of the empirically derived interaction vectors. According to vector analysis, the constructed vector interaction field can be used to infer a scalar potential field with the help of vector decomposition and integration. The inferred scalar potential acts as a forcing function that induces the interaction flows. Then, given this forcing function, Gastner and Newman (2004)'s diffusion-based method can be applied to produce a density-equalizing areal cartogram, which serves as a global representation of the reconstructed migration space.

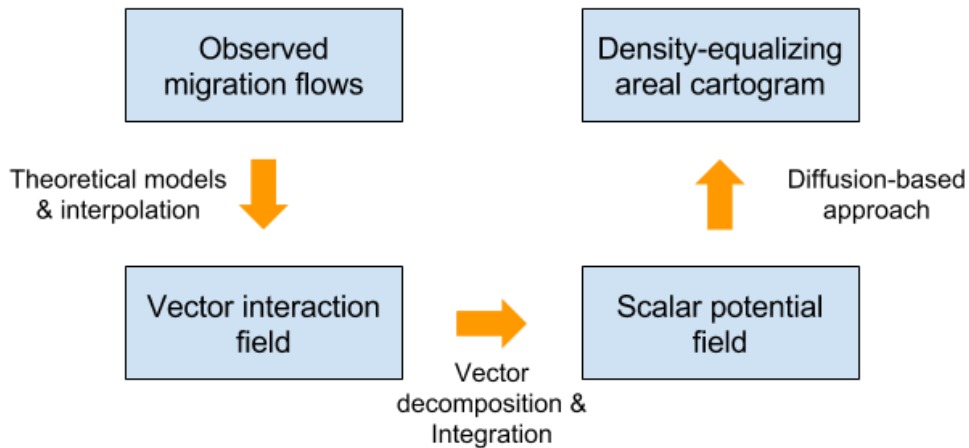


Figure 1 Flow chart for the procedural methods

3. Preliminary results

For demonstration purpose, preliminary results have been obtained and presented as follows. Figure 2(a) illustrates the empirically derived interaction vectors based on the model in section 2.1. U.S. interstate migration data for the year of 2000 from the U.S. Census is used as a use case. Blue dots represent the population weighted centroids for all the 48 continental states. Figure 2(b) shows a vector interaction field after interpolating the empirical interaction vectors in Figure 2(a). This interaction field depicts a general pattern that migration flows are facilitated from middle areas to

Northeast, west coastal areas, and some part of the Midwest areas.

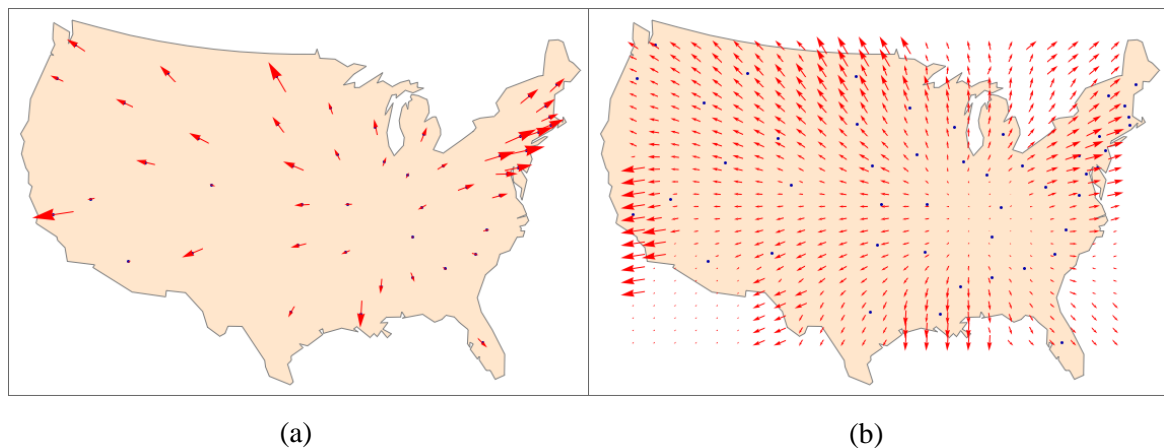


Figure 2 Migration vectors (a) and interpolated vector field (b)

4. Conclusive discussions

This study aims to produce a new methodological approach to a global representation of functional space derived from empirical spatial interaction data. By linking spatial interaction models with the concepts of vector field and the inferred potential field, this approach enables the application of areal cartograms to represent the reconstructed functional space, which may convey information via a more intuitive perspective than the MDS-based coordinate systems and linear cartograms previously used. The key to a better understanding of the global picture of a functional space in question is how to synthesize the flow vector \vec{c}_{ij} in this approach. In addition to the most straightforward way (Equation 7) used by Tobler (1967), there are other opportunities to be explored inspired by theories of vector analysis.

Biography

Zhaoya Gong is a Lecturer in Human Geography at the School of Geography, Earth and Environmental Sciences at the University of Birmingham. His research centers on leveraging computational and data sciences to advance GIScience, and dynamic processes of urbanisation and complex modalities of spatial structures generated at various scales.

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