Quantifying Movement Probabilities in Space-Time Prisms using Cost-Path Analysis

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Summary

Predicting movement between two locations remains a challenge in GIScience. Many models have been developed based on random walks, but these models fail to account for structural heterogeneity that impedes movement potential. Here, a model is developed for estimating movement probabilities between two points combining ideas from Hägerstrand's time geography and popular GIS-based cost-path analysis tools. The outcome is a model for estimating movement probabilities in space-time prisms where movement occurs across a heterogeneous spatial lattice. The application of the model in the field of movement ecology is likely to offer new potential for studying animal space-use patterns.

KEYWORDS: Movement, time geography, spatial field, accessibility, random walk.

1 Introduction

In recent years there has been considerable development and application of the core ideas stemming from Hägerstrand's (1970) seminal framework of time geography. However, one of the main limitations of time geography, and specifically the use of space-time prisms, is that by definition space-time prisms are discrete, providing only the outer boundary of movement opportunity. Thus, new developments have attempted to quantify the unequal movement probabilities within space-time prism — that is the inner structure of space-time prisms. The most well developed ideas were originally proposed by Winter and Yin (2010, 2011) who framed the problem in the context of random walks, termed probabilistic time geography. Song and Miller (2014) extended the ideas from probabilistic time geography to incorporate a more robust statistical framework — namely combining time geography with popularized Brownian bridge models.

Prior to these developments, Miller and Bridwell (2009) proposed the conceptual framework for a *field-based time geography*, which represents a more pragmatic approach to modelling the unequal movement possibilities within space-time prisms. Field-based time geography considers the potential

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movement limitations as a spatial field which serves as the basis for calculating internal movement possibilities. Miller and Bridwell (2009) discuss the potential of this approach both in the context of movement along a spatial network and across a spatial lattice (i.e., a cost surface). In their example a transportation network is used to demonstrate the non-regular shapes of resulting from consideration of variable movement speeds in the construction of network space-time prisms, and this approach is now and has been commonly used in the generation of *isochrones* (lines of equal travel time). A simple example of a lattice application is provided on synthetic data, but not further developed.

The objective of this paper is three-fold: 1) to formalize the definition of field-based time geography for applications on heterogeneous spatial lattices, 2) to develop models for estimating probabilities of movement across a heterogeneous spatial lattice, and 3) provide the derivation of an algorithm for field-based time geography, along with an implementation in a free and open source software environment. Synthetic data is used to highlight the novel interpretations gained from field-based time geography. Discussion will centre on the development of the model, rather than on practical inferences.

2 Methods

2.1 Field-based time geography

The construction of field-based time geography follows classic time geography by considering the intersection of space-time cones. In order to do so consider any intermediate time point t between two anchors a and b, where $t_a < t < t_b$. For a location (typically a pixel) we define two accumulated cost surfaces: T_{ai}, which is the cost (in units of time) from the location a to location i based on the network N (similarly compute T_{ib}). If location i is accessible at time t (i.e., $T_{ai} \leq t - t_a$; and $T_{ib} \leq t_b - t$) then location i is within the potential path space at time t (PPS_t).

Next, the deviation of each of each of the T_{ai} and T_{ib} from the *expected* time budget which is the associated with travelling along the *least cost path* is calculated.

$$\Delta T_{ai}(t) = \left| T_{ai} - \frac{t - t_a}{t_b - t_a} T^*_{AB} \right| \tag{1}$$

$$\Delta T_{ib}(t) = \left| T_{ib} - \frac{t_b - t}{t_b - t_a} T^*_{AB} \right| \tag{2}$$

Where T^*_{AB} is least cost path time from A to B. The two time deviations (from the forward and past cones) are summed to compute the overall time deviation for each location i at time t:

$$\Delta T_i(t) = \Delta T_{ai}(t) + \Delta T_{ib}(t) \tag{3}$$

The location i associated with travel along the least cost path at time t will have $\Delta T_i(t) = 0$.

With field-based time geography we are interested in estimating the *probability* an object was at a location at a given time — $\hat{P}_{it}$. Thus, we must define a function to transform the overall time deviations ($\Delta T_i(t)$) from equation (3) into probability values.

$$\hat{P}_i(t) \propto \; f(\Delta T_i(t)) \tag{4}$$

There are, however, many potential mathematical functions that we could use to define $\hat{P}_i(t)$, see for example Table 1 which is developed after Taylor (1975) and Haggett et al. (1977). The most straightforward way to model movement probabilities in the field-based space-time prism is to estimate the probability the individual visited location i at time t as proportional to the inverse of the travel-time. However Haynes et al. (2003) discusses the growing trend to use inverse-squared functions, typically in spatial interaction models. Alternatively, negative exponential functions have the firmest theoretical foundation for modelling the decreasing activities as a function of distance, cost or time (Haynes et al., 2003; Handy and Niemeier, 1997; Wilson, 1967). Each of these models underscored by the notion that movement will typically follow the route of least resistance (Haggett et al., 1977) which is based on the *principle of least effort* (Zipf, 1949).

Table 1: Potential functions for modelling movement probability as a function of time (taken from distance decay functions commonly used in spatial interaction models; Taylor, 1975; Haggett et al., 1977). The constant c is a stabilizing parameter for when t is close to 0 in the inverse models and a tuning parameter in the exponential family of models.

Model	Function
Inverse	$\frac{1}{t+c}$
Inverse^2	$\frac{1}{(t+c)^2}$
Exponential	e^{-ct}
Normal	e^{-ct^2}
Root Exponential	$e^{-c\sqrt{t}}$
Pareto	$e^{-c\log t}$
Log Normal	$e^{-c\log t^2}$

It is necessary to standardize the $\hat{P}_i(t)$ so that $\sum P_i(t) = 1$ for any time t in order to account for variations in the size of the PPS_t (see Winter and Yin, 2011; Song and Miller, 2014). This can be done simply by dividing the $\hat{P}_i(t)$ by the sum of all the $\hat{P}_i(t) \in$ the PPS_t:

$$P_i(t) = \frac{\hat{P}_i(t)}{\sum_{\forall j} \hat{P}_j(t)}, \quad j \in PPS_t \tag{5}$$

The $P_i(t)$ can be used to study the internal movement probabilities within field-based space-time prisms. Several types of further analysis to allow the $P_i(t)$ toe be analyzed more practically. First, a map of the $P_i(t)$ for any given t can be used to quantify movement potential at a specific time. Both Winter and Yin (2010) and Song and Miller (2014) use incremental maps of the PPS_t to demonstrate how the $P_i(t)$ change through time within a space-time prism. Such a mapping is useful to visualize and analyze the potential movement probabilities at a particular time.

The cumulative visit probability (P_i) for any location i over the entire time interval between t_a and t_b is defined as:

$$P_i = \int_{t_a}^{t_b} P_i(t)\, dt \tag{6}$$

In practice, the integral in equation (6) is not easy to calculate, but we can approximate it by taking a set of equally spaced times between t_a and t_b (i.e., $t_a < t_k < t_b$) and performing numerical integration using the trapezoid rule. The map of the P_i for the entire space-time prism represents the probabilistic version the potential path area — the projection of the space-time prism onto the spatial plane. For any space-time prism the sum of the P_i is equal to the time budget of the prism, that is $\sum P_i = t_b - t_a$. This definition of P_i is extremely powerful because it facilitates easy interpretation of modelled probabilities relative to the overall time budget.

2.2 Three example scenarios

Three example scenarios of movement between two points (A and B) are used to demonstrate the new field-based time geography model for estimating movement probabilities within the space-time prism. The first scenario represents the case where the anchor location B is situated in a low conductance area, for example walking up a hill. The second scenario represents the case where a circular barrier (low conductance) is situated between the anchors A and B. The third scenario represents a more realistic case, movement through a heterogeneous environment. The introduction of environmental data into wildlife movement analysis is an active area of research, and the model developed here can be readily applied using commonly available data (e.g., remotely sensed land cover, DEMs) which can be related to restricted movement in terrestrial animals.

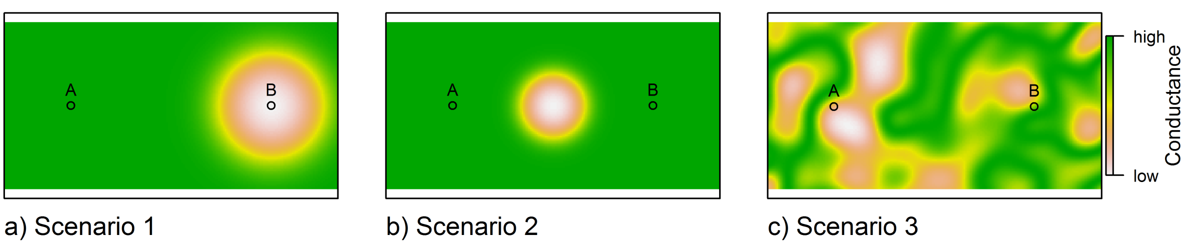

Figure 1: Three scenarios used to examine field-based time geography a) walking up a hill, b) a circular barrier, c) a heterogeneous landscape.

3 Results & Discussion

In figure 2, the inverse time-function is used to model movement probabilities within field-based space-time prisms. The shape of the underlying probability surface reflects the presence of the low conductance areas in figure 1. Here we can see that with field-based space-time prisms, which are based on cost-path analysis, the most probable movement path follows the 'least cost path' (shown as a dotted line).

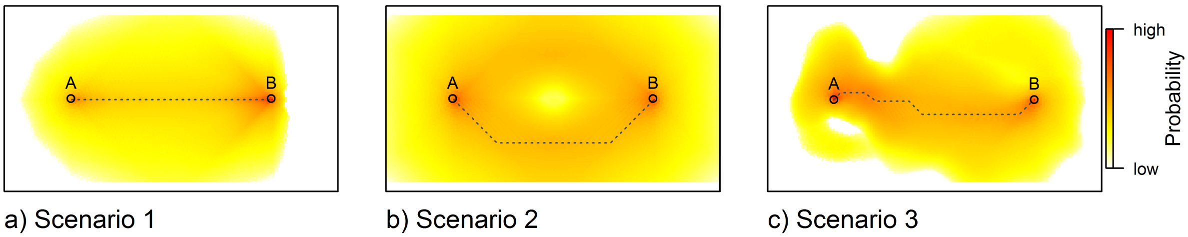

Figure 2: Output probability surfaces for the three scenarios from figure 1. The dotted line shows the least cost path between anchor points A and B.

The presentation of this paper will examine how the various time functions displayed in Table 1 influence the resulting probability surfaces. From this analysis, the goal is to develop a framework for a future study that will test these movement probabilities based on controlled cases with high resolution tracking data (for example using orienteering as a case study Kay, 2012). This future work may open up opportunities for studying human spatial cognition and wayfinding optimization strategies in order to minimize resistance (i.e., time or effort) across a spatial lattice. This analysis would lend empirical support for the principle of least effort (Zipf, 1949) and whether or not humans are able to optimize spatial movement based on different types of knowledge of their environment (e.g., maps, signage, mobile-application data) and local vs. global information.

The use of cost-path analysis represents an important development in time geographic analysis as all previous models for estimating movement probabilities within space-time prisms have focused on random walks which assume the bee-line is the most probable movement path between anchor points A and B (Winter and Yin, 2010). The assumption of random movement is unrealistic in most applications, and thus existing approaches fail to adequately consider underlying spatial heterogeneity that shapes movement. The biggest potential application of this model is to derive improved measures of animal space use, commonly refereed to as the home range or utilization distribution (Worton, 1989). Through the use of field-based time geography, ecologists may be able to improve and refine home range estimates and better prioritize areas for conservation; reflecting how underlying landscape heterogeneity impacts movement.

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Biography

Jed Long is a Lecturer in GeoInformatics in the School of Geography & Sustainable Development at the University of St Andrews. His research interests span quantitative geographical analysis and spatial ecology. Much of his work focuses on studying spatial patterns in wildlife movement through the use of GPS tracking data.

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